

# **Can Thin Film Alloy Refractive Index Be Accurately predicted?**

**---Theoretical & Experimental Study Thin Film Alloy AgZn Refractive Index**

**By Daniel Lin, Tevin Ding and Guowen Ding  
LabForInvention Corp**

# Abstract

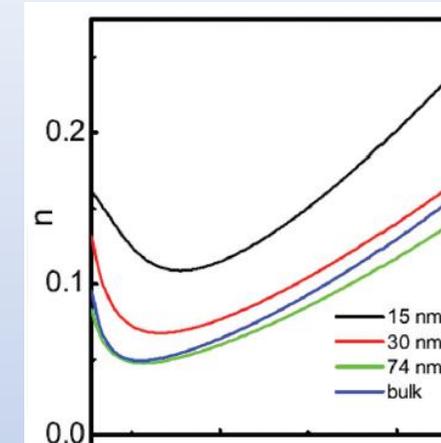
- The optical refractive index and electrical resistivity  $\rho$  of Silver Zinc alloy were studied experimentally by a co-sputtering method.
- Currently, the theories on alloys refractive index are too complex, dependent on too many factors, so it can still only be a semi-practical prediction.
- In this study, a new theoretical method was developed to simplify the calculation of silver alloy IR refractive index, and also valid experimental condition was found.
  - (1) Theoretically, the refractive index ratio  $n_{\text{alloy}}/n_{\text{Ag}}$  between silver and its alloy at IR region is derived, a way to cancel out most factors so that it only depends on free electron density  $n_e$  and the film resistivity  $\rho$
  - (2) Experimentally, this assumption was approximately met at a special conditions
  - (3) The experimental results agreed well with this simplified calculation.
- Further how the zinc electrons contribute to the optical and electrical properties in low Zn concentration AgZn alloy (<10%) was experimental presented and discussed in this study.

# Introduction

- We are initially curious
  - Why are there no accurate theory calculations for metal thin film refractive index?
- Can we derive such an accurate theoretical calculation?
- Can we prove the theory by experiments?

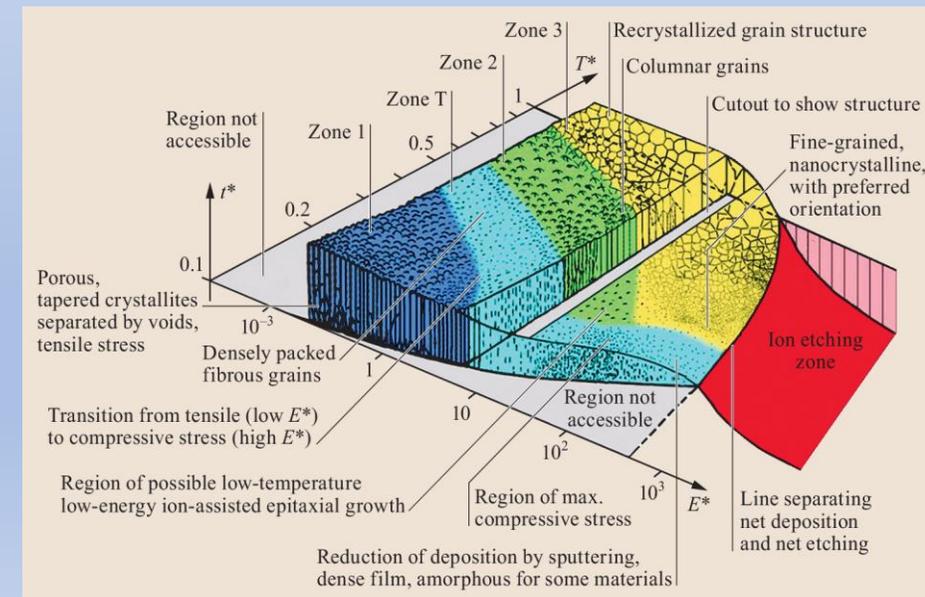
# Why refractive index calculation is hard to meet thin film value

- The thin film refractive index is dependent on too many factors,
  - up to multiple hundred % impacting on the index
- Complexity physics for thin film alloys optical and electrical properties
  - Alloy Atom structure, Crystal or amorphous/Crystal structure
  - Crystal defects/ Materials impurity
  - Film thickness / roughness / Grain size
  - Substrate materials
  - Ambient environmental conditions (temperature, humidity, queue time)
  - Deposition condition: pressure/ temperature / power / gun to substrate distance/...



Ref. 3  
silver thin film  
refractive index  
varied more than 2  
times

Thornton diagram: thin film properties varied significantly as different deposition conditions



# Theory on metal properties calculation

Free electron model Assumption: (function of  $X$ ,  $E$ (electric field),  $\omega$  (frequency),  $\tau$ (collision time))

Dielectric Constant  $\epsilon_r$  definition  
(function of  $X$ ,  $E$ )

Dielectric Constant  $\epsilon_r$  is  
function of  $\omega$ ,  $\tau$ ,  $\omega_{pe}$  frequency,  
collision time, plasma frequency, which is  
function of electron density ( $n_e$ )

optical and electric properties

More  
In detail

- Drude-Lorentz Model assumptions: (widely used, especially for group I and XI elements)

- In math:  $-eE(t) = m_e \frac{d^2x}{dt^2} + \frac{m_e}{\tau} \frac{dx}{dt}$  (1)

- The Solution is  $X = \frac{eE}{m_e(\omega^2 + i\frac{\omega}{\tau})}$  (2)

- The electric displacement field:  $D \equiv \epsilon_0 E + P \equiv \epsilon_r \epsilon_0 E$  (3)

- And polarization  $P = -e \cdot n_e x$  (4)

- Substitute Eqs.(2),(4) into Eq. (3)  $\rightarrow$   
dielectric constant  $\epsilon_r = 1 - \frac{\omega_{pe}^2}{\omega^2 + i\frac{\omega}{\tau}} = 1 - \frac{\omega_{pe}^2 \tau^2}{1 + \omega^2 \tau^2} + i \frac{\omega_{pe}^2 \tau^2}{\omega \tau (1 + \omega^2 \tau^2)}$  (5)

Where  $\omega_{pe}$  is plasma frequency,  $\omega_{pe}^2 = \frac{n_e e^2}{\epsilon_0 m_e}$  (6)

- And Ohm's & Newton's laws  $\rightarrow$  Collision time  $\frac{1}{\tau} = \frac{n_e e^2 \rho}{m_e}$  (7)

e: is electron charge  
 $m_e$  is electron mass  
 $\tau$ : is the collision time  
 $\omega$ : is frequency  
 $\omega_{pe}$ : is plasma frequency  
 $n_e$  is electron density  
 $E$ : electric field  
 $D$ : electric displacement field  
 $P$ : polarization  
 $\epsilon_0$ : is dielectric Constant in vacuum  
 $\epsilon_r$ : is relative dielectric Constant  
 $\epsilon_1$ : is real part of dielectric Constant  
 $\epsilon_2$ : is imaginary part of  $\epsilon_r$   
 $\rho$ : is resistivity

# Refractive Index calculation was simplified from literatures

- Refractive index mathematically

$$n + ik = \sqrt{\epsilon_r} = \sqrt{\epsilon_1 + i\epsilon_2}$$

Calculation Eq(5) can be simplified by

- Assumption 2:**  $|\epsilon_1| \gg |\epsilon_2|$ ;

– Valid for group 1 and 11

- Assumption 3:**  $\omega^2 \tau^2 \gg 1$ ;

– Valid for wavelength  $< 3\mu m$

→ index  $n$  is function of

– Electron density  $n_e$

– Resistivity  $\rho$ ,

– Frequency  $\omega$

$$\epsilon_r = 1 - \frac{\omega_{pe}^2}{\omega^2} + i \frac{\omega_{pe}^2}{\omega^3 \tau} \quad (15)$$

$$n = \frac{n_e^2 e^4}{\epsilon_0 m_e^2} * \rho * \frac{1}{2k\omega^3} \quad (16)$$

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba *	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn

$n$  is common refractive index  
 $k$  is extinction coefficient  
 $e$ : is electron charge  
 $m_e$  is electron mass  
 $n_e$ : is electron density  
 $\epsilon_0$ : is dielectric Constant in vacuum  
 $\rho$ : is resistivity

# Develop a method for index calculation with two factors

- **Further simplification by an assumption 4:  $\omega_{pe}^2 \gg \omega^2$**

- valid for wavelength > 0.6 um

- $\epsilon_r$  can be simplified as Eq. 17, then substitute Eq. 18 into Eq. 16

$$\epsilon_r = -\frac{\omega_{pe}^2}{\omega^2} + i\frac{\omega_{pe}^2}{\omega^3\tau} \quad (17)$$

$$k = \sqrt{|\epsilon_1|} = \sqrt{\frac{\omega_{pe}^2}{\omega^2}} = \frac{\sqrt{\epsilon^2 n_e}}{\epsilon_0 m_e \omega} \quad (18)$$

- **Ratio  $\rightarrow$  cancel out most factors in index calculation**

**Index ratio**

$$\frac{n_{alloy}}{n_{Ag}} = \frac{n_{e-alloy}^{3/2} * \rho_{alloy}}{n_{e-Ag}^{3/2} * \rho_{Ag}} \quad (21)$$

$$\text{silver index } n_{Ag} = \frac{n_{e-Ag}^{1.5} e^3}{2\sqrt{\epsilon_0} m_e^{1.5} \omega^2} * \rho_{Ag} \quad (19)$$

$$\text{alloy index } n_{Alloy} = \frac{n_{e-Alloy}^{1.5} e^3}{2\sqrt{\epsilon_0} m_e^{1.5} \omega^2} * \rho_{alloy} \quad (20)$$

only dependent on two factors  $n_e$  and  $\rho$ , (free electron density and resistivity), with all other factors being cancelled out: (Assumption 5)

$n_{Ag}$  : is silver index  
 $n_{Alloy}$  : is alloy index  
 $\rho_{Ag}$  : is silver resistivity  
 $\rho_{alloy}$  : is alloy resistivity  
 $n_{e-ag}$  : is silver electron density  
 $n_{e-alloy}$  : is alloy electron density

- **Can experiments be valid for these two factors? Yes, under special conditions**

- The Ag and AgZn alloy with low Zn concentration (<10%): The alloy co-sputter deposition conditions nearly identical to Ag's condition, except the alloy having an additional tiny Zinc co-sputtering power.

# Derive thin film alloy refractive index calculation

- The alloy index can be calculated by  $n_e$  and  $\rho$  (eq. 21)
- Electron densities  $n_e$  of alloys have 2 possibilities
  - Zn atoms contributes 0 free electrons;

$$\text{Alloy Index } n_{\text{alloy}} = (1 - \text{Zn}\%)^{3/2} * \frac{\rho_{\text{alloy}}}{\rho_{\text{Ag}}} * n_{\text{Ag}} \quad (23)$$

- Each Zn atom contributes 2 free electrons;

$$\text{Alloy Index } n_{\text{alloy}} = (1 + 1.24\text{Zn}\%)^{3/2} * \frac{\rho_{\text{alloy}}}{\rho_{\text{Ag}}} * n_{\text{Ag}} \quad (25)$$

- Experiments can tell which is the correct one
- The alloy index can be accurately predicted if Ag index, Ag-Alloy resistivity and concentration are known

electron density calculation

$$\text{Alloy electron density } n_{e_{\text{alloy}}} = (1 - \text{Zn}\%)n_{e_{\text{Ag}}}$$

Alloy electron density:

$$\begin{aligned} n_{e_{\text{alloy}}} &= (1 - \text{Zn}\%)n_{e_{\text{Ag}}} + 2 * \text{Zn}\% * n_{e_{\text{Zn}}} \\ &= (1 - \text{Zn}\%) n_{e_{\text{Ag}}} + 2 * \text{Zn}\% \left( \frac{7.13}{10.49} \frac{107.87}{65.38} n_{e_{\text{Ag}}} \right) \end{aligned}$$

$n_{e_{\text{Zn}}}$  is transferred to  $n_{e_{\text{Ag}}}$  by

the molecular weight and density conversion

Where Zn% is the volume percentage of Zn in the alloy

Silver molecular weight: 107.87 u

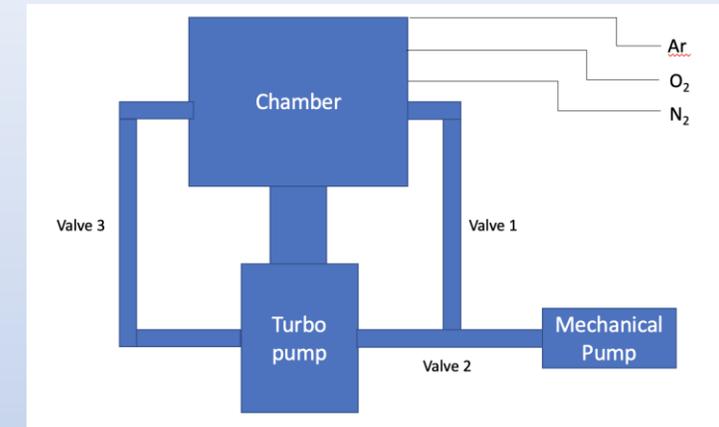
Zinc molecular weight: 65.38 u

Silver density: 10.49 g/cm<sup>3</sup>

Zinc density: 7.13 g/cm<sup>3</sup>

# Experiment Setup

- PVD
  - Background vacuum:  $2 \times 10^{-7}$  torr
  - Ag and Zn target materials are 99.99%
  - Co-sputter to generate Ag Zn alloy
  - Pulsed DC (40kHz) power supply
- UV-VIS-IR Spectrometer (Shimadzu 3700)
  - 300-2500 nm spectra
  - Double beams, 3 sensors
  - Error bar is 0.2%
- Spectroscopic Ellipsometer (Woollam VASE)
  - 300-1700 nm
  - 3 measurement parameter (Delta, Psi, transmission)
  - Thickness accuracy 0.1 nm
- Four Point Probe
  - Accuracy 0.04 ohm



# Zn contributes free electrons in alloy AgZn properties or not?

- If each Zn atom contributes 2 free electrons in alloy properties

$$\frac{n_{\text{alloy}}}{n_{\text{Ag}}} / \frac{\rho_{\text{alloy}}}{\rho_{\text{Ag}}} = (1 + 1.24\text{Zn}\%)^{3/2} \quad \times \quad (25)$$

- If each Zn atom contributes 0 free electrons in alloy properties

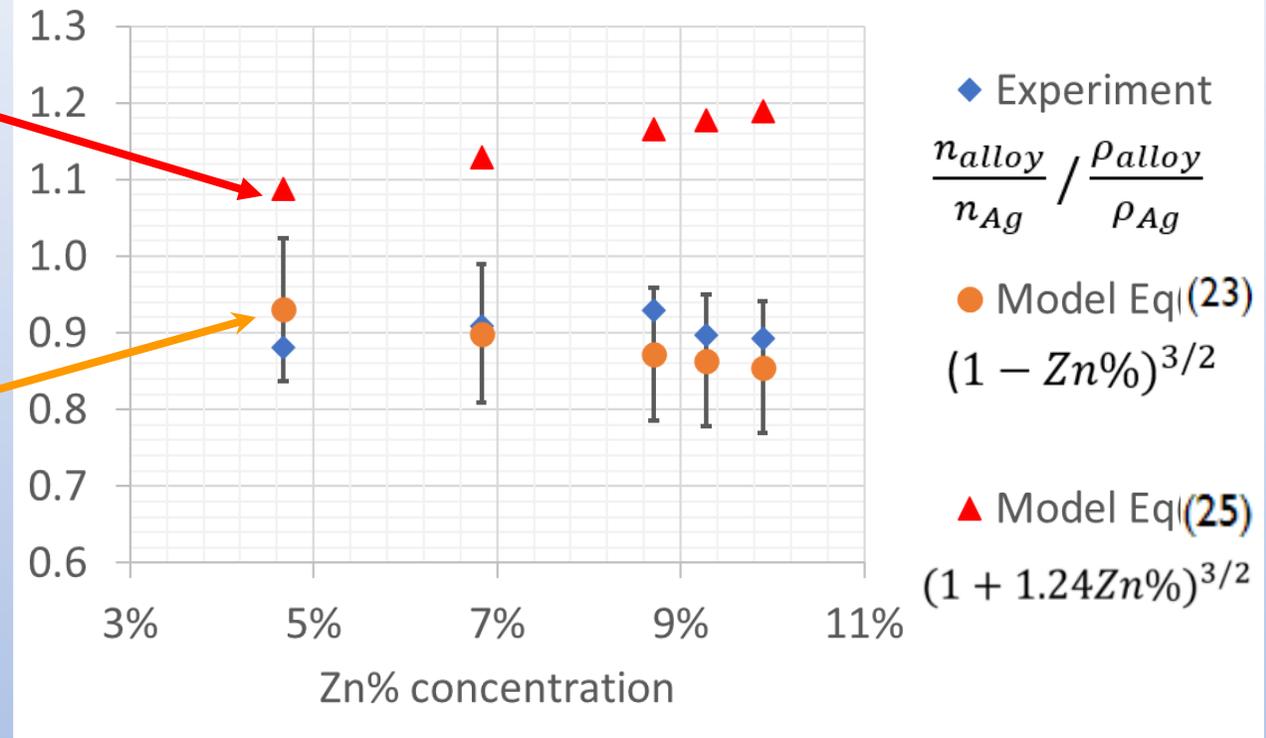
$$\frac{n_{\text{alloy}}}{n_{\text{Ag}}} / \frac{\rho_{\text{alloy}}}{\rho_{\text{Ag}}} = (1 - \text{Zn}\%)^{3/2} \quad \checkmark \quad (23)$$

- In conclusion, Zn contributes no free electrons

- Error bar discussion.  $< \pm 12\%$

- Calculated from the sum of all standard deviations of measurements of  $\rho$  ratios  $< 5\%$
- plus the deposition  $n_{\text{Ag}}$  and  $n_{\text{alloy}}$  ratio variation of  $< 7\%$ ,

Model vs experiment results comparison



# Runsheets Summary

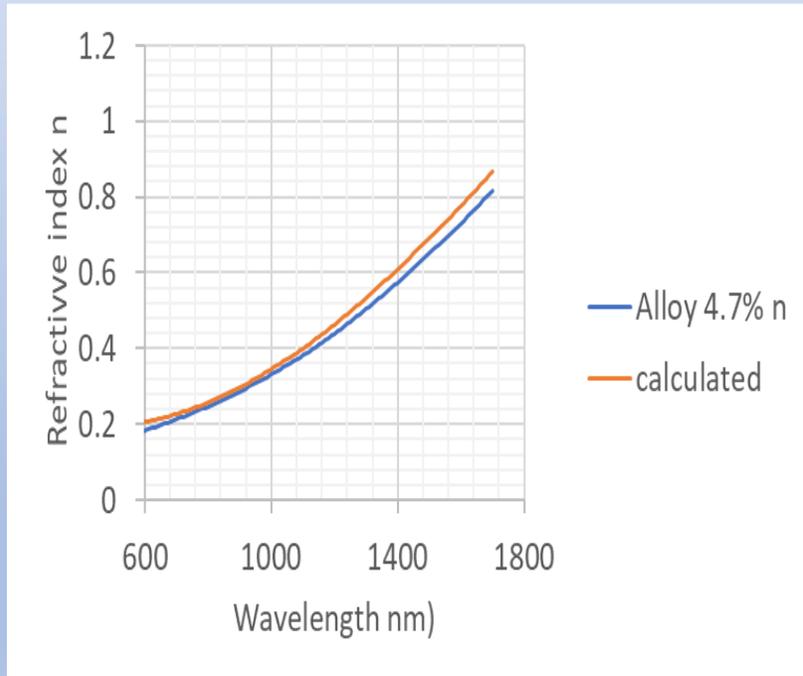
run #	Zinc power	Ag power	pressure (mT)	Ar flow (sccm)	Time Sec	thickness (nm)	Dep Rate (nm/s)	average Rs (ohm)	SteEV	1sigma	Resistivity (ohm*m)	Zn% in volume	alloy electron density (/m3)	collision time $\tau$ (s)	wavelength $\lambda$ for $(\omega\tau)^2 \gg 1$ (nm)	plasma frequency (Hz)	wavelength $\lambda$ (nm) for $(\omega_{pe}/\omega)^2 \gg 1$
D71-3		100w	2.5	360	70	26.4	0.38	1.4	2.5%	1.9%	3.6E-08	0	5.80E+28	1.71E-14	1.02E+04	1.36E+16	4.38E+02
D71-7	6w	100W	2.5	360	70	29.1	0.42	3.8	4.5%	1.2%	1.1E-07	9.3%	5.26E+28	6.09E-15	3.63E+03	1.29E+16	4.60E+02
D72-1		150W	2.5	360	60	34.1	0.57	1.0	2.2%	2.2%	3.4E-08	0	5.80E+28	1.80E-14	1.07E+04	1.36E+16	4.38E+02
D72-3	6w	150W	2.5	360	60	36.6	0.61	2.7	2.8%	1.0%	9.9E-08	6.8%	5.40E+28	6.65E-15	3.96E+03	1.31E+16	4.54E+02
D72-5		200W	2.5	360	45	34.7	0.77	1.1	1.9%	1.8%	3.7E-08	0	5.80E+28	1.66E-14	9.91E+03	1.36E+16	4.38E+02
D72-7	6w	200W	2.5	360	45	36.4	0.81	2.2	2.0%	0.9%	7.9E-08	4.7%	5.53E+28	8.10E-15	4.82E+03	1.33E+16	4.49E+02
D73-1		100W	2.5	360	90	34.6	0.38	1.0	3.0%	3.0%	3.5E-08	0	5.80E+28	1.77E-14	1.05E+04	1.36E+16	4.38E+02
D73-5	6w	100W	2.5	360	90	37.9	0.42	2.9	2.7%	0.9%	1.1E-07	8.7%	5.29E+28	6.19E-15	3.69E+03	1.30E+16	4.59E+02
D73-7	6w	100W	2.5	360	90	38.4	0.43	2.8	4.4%	1.5%	1.1E-07	9.9%	5.23E+28	6.26E-15	3.73E+03	1.29E+16	4.62E+02

- 3 different Zn% of AgZn alloy were studied by co-sputter method,
- To meet our assumptions, we need the wavelength to be at least greater than 600nm and smaller than 3000nm:
  - Assumption 3:  $\omega^2 \tau^2 \gg 1$ ; Valid for wavelength  $< 3\mu m$
  - Assumption 4:  $\omega_{pe}^2 \gg \omega^2$  : valid for wavelength  $> 0.6 \mu m$

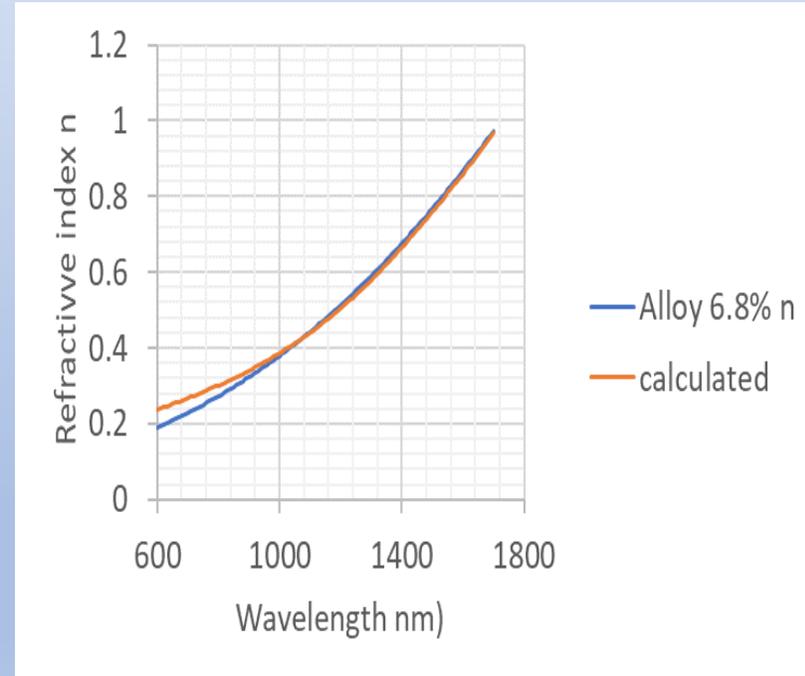
# Alloy index model calculation vs experiment measurements

$$\text{Alloy Index : } n_{\text{alloy}} = (1 - \text{Zn}\%)^{3/2} * \frac{\rho_{\text{alloy}}}{\rho_{\text{Ag}}} * n_{\text{Ag}} \quad (26)$$

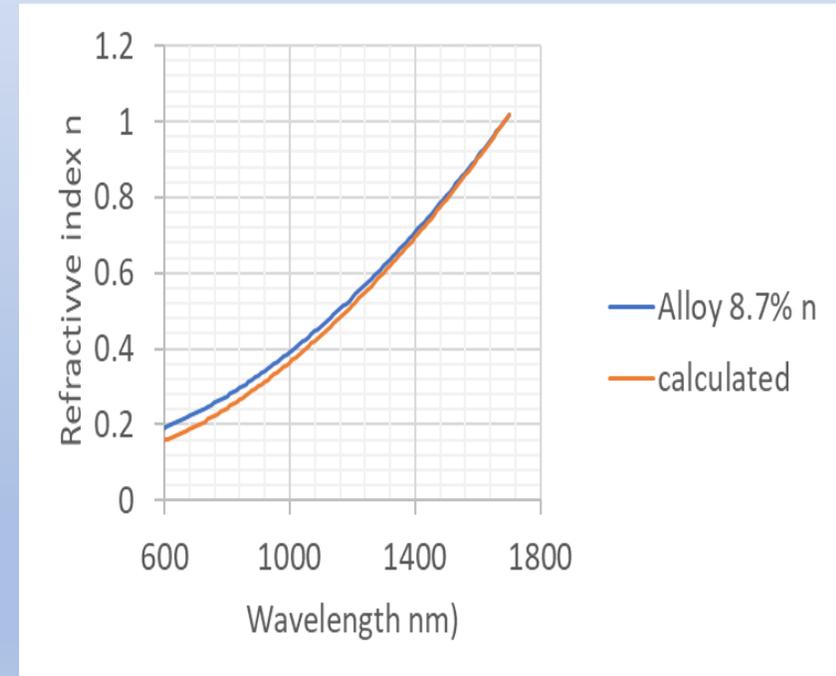
AgZn (Zn 4.7%)



AgZn (Zn 6.8%)



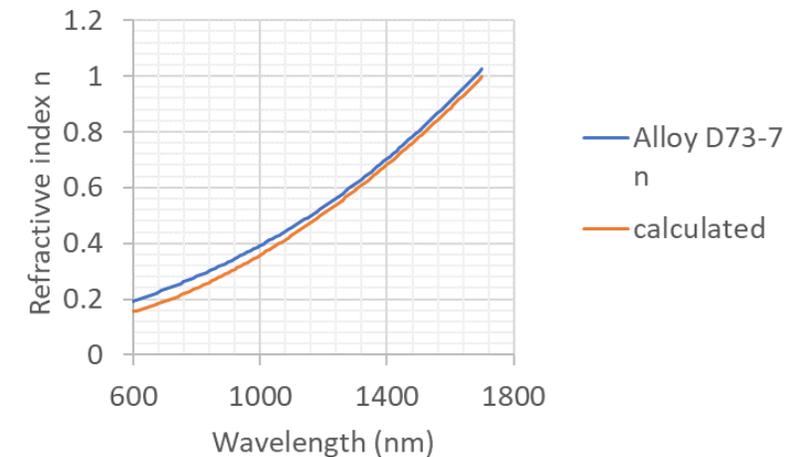
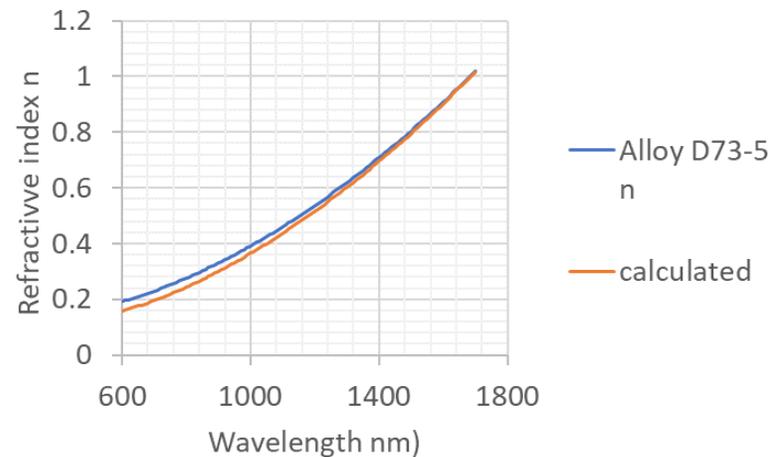
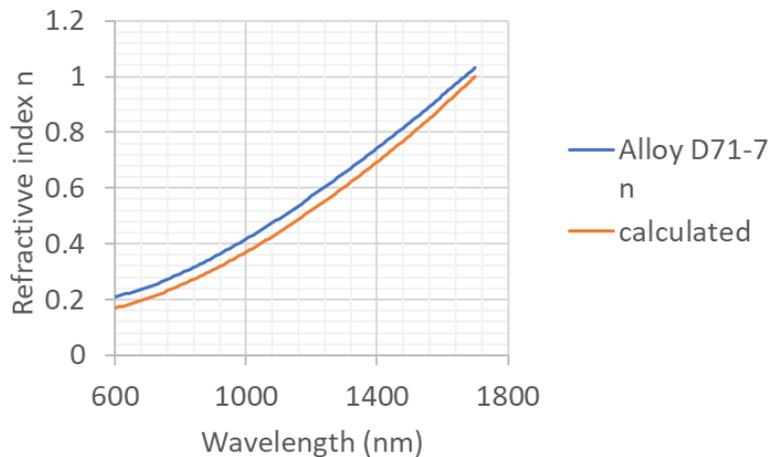
AgZn (Zn 8.7%)



The model calculations and the experiment results match up well

# Repeatability study:

- Indexes ratio variation from three model calculation showed  $\sim 7\%$  standard deviation in average of the spectra.
  - Two Silver depositions of 100W with  $<4\%$  index variation
  - Three Alloy depositions of 100W Ag + 6 W Zn with  $<3\%$  index variation



Repeated Silver alloy index of 100wAg + 6W Zn and the calculation from the theory eq. (26)

# Conclusions

- We theoretically derived a refractive index calculation that depends only on two parameters which canceled out other factors.

$$\frac{n_{alloy}}{n_{Ag}} = \frac{n_{e-alloy}^{3/2} * \rho_{alloy}}{n_{e-Ag}^{3/2} * \rho_{Ag}}$$

- Valid experimental conditions were confirmed between 0.6  $\mu\text{m}$  and 3  $\mu\text{m}$
- Alloy index with concentration relationship was derived:
$$n_{alloy} = (1 - Zn\%)^{3/2} * \frac{\rho_{alloy}}{\rho_{Ag}} * n_{Ag}$$
  - Theory and experiments agree within error bar
  - Experimentally confirmed that Zn contributes no free electrons in alloy AgZn film properties in this study.

# Future study on why Zn does not contribute free electron in AgZn alloy?

- This study indicated that Zn does not contribute free electron in AgZn alloy.
- Literature report AgGe with alloy concentration 26%-37%, the 2<sup>nd</sup> element Ge contribute free electrons
- However, there are literature of AgFe with 1%, AlCu(0.5%-1.5%) ....alloy free electrons are lower than the pure silver concentration, which means the 2<sup>nd</sup> element is likely not contribute free electron.
- Our hypothesis is: at very low concentration of alloy element in a highly conducting metal silver, the electrons from the 2<sup>nd</sup> element is localized and is not able to contribute any free electrons to the conducting current.
- Any suggestion?

# References

1. Mark Fox, "Optical Properties of Solids", 2012, 2nd edition, Oxford University Press Inc. New York
2. H. G. Tompkins, W, A, McGahan, "Spectroscopic Ellipsometry and Reflectometry", 1999, A wiley-Interscience Publication, New York.
3. G. Ding, C. Clavero, D. Schweigert, M. Le, "Thickness and microstructure effects in the optical and electrical properties of silver thin films" American Institute of Physics ADVANCES, Volume: 5 Issue: 11 Article Number: 117234, NOV 2015
4. V NGUYEN VAN, S. FISSON AND M -L THEYE, "OPTICAL AND ELECTRICAL INVESTIGATIONS OF AMORPHOUS Ag-Ge METALLIC ALLOY FILMS", *Thin Solid Films*, 89 (1982) 315-321
5. Chadrsekhar Loka, Kee-Sun Lee, "Reflectance changes of Fe and Cr doped Ag thin films deposited by magnetron sputtering", *Thin Solid Films* 641 (2017) 73–78
6. A.A. Wronkowska \*, A. Wronkowski, K. Kuklin´ski, M. Senski, Ł. Skowron´ski, "Spectroscopic ellipsometry study of the dielectric response of Au–In and Ag–Sn thin-film couples", *Applied Surface Science* 256 (2010) 4839–4844
7. Guang Yang, Xiao-Jian Fu, Jing-Bo Sun, Ji Zhou "Spectroscopic ellipsometry study on the optical dielectric properties of silver platinum alloy thin films", *Journal of Alloys and Compounds* 551 (2013) 352–359
8. Lance W. Barron, Jason Neidrich, Santosh K. Kurinec, L.W. Barron et al. "Optical, electrical, and structural properties of sputtered aluminum alloy thin films with copper, titanium and chromium additions", *Thin Solid Films* 515 (2007) 3363–3372
9. A.A. Wronkowska et al. "Optical and microstructural characterization of Au–Sn and Cu–Sn diffusive layers", *Applied Surface Science* 281 (2013) 30– 37

Please leave your contact information for a poster copy